32.18 A parallel flow heat exchanger cools $500gpm$ of water from $65°F$ to $55°F$ with $700gpm$ of a 50% ethylene glycol mixture ($\rho = 67.5\frac{lb_m}{ft^3}$, $c_p = 0.765\frac{Btu}{lb_m \cdot °F}$) entering at $32°F$. The overall coefficient of heat transfer for the heat exchanger is $60\frac{Btu}{hr \cdot ft^2 \cdot °F}$. What is the heat transfer surface area required?

A. $1900ft^2$

B. $2900ft^2$

C. $4200ft^2$

D. $4800ft^2$

The heat supplied by the warm fluid, water, is added to the glycol mixture. Set these quantities of heat equal. This quantity is also equivalent to the total heat transfer for the heat exchanger, which is based on the Log Mean Temperature Difference.

$$\dot{Q}_{water} = \dot{Q}_{glycol} = \dot{Q}_{hx}$$

The heat supplied by the water can be expressed using the sensible heating/cooling rule of thumb for water and will have units of $\frac{Btu}{hr}$.

$$\dot{Q}_{water} = 500gpm\Delta T = 500\,(500)\,(65-55) = 2.5 \times 10^6 \frac{Btu}{hr}$$

The glycol mixture has a different density and specific volume than water, so it is necessary to use the more general formula for sensible heating/cooling of any substance.

$$\dot{Q}_{glycol} = \dot{m}c_p\Delta T$$

Recall that mass flow rate is the product of density and volume flow rate. Substitute and adapt units, solving for the exit temperature of the glycol mixture. Show all units since this is not a "rule of thumb" formula.

$$\dot{m} = \rho Q$$

$$\dot{Q}_{glycol} = \rho Q c_p \Delta T$$

$$\dot{Q}_{glycol} = \left(67.5\frac{lb_m}{ft^3}\right)\left(700\frac{gal}{min}\right)\left(\frac{1ft^3}{7.48gal}\right)\left(\frac{60min}{1hr}\right)\left(0.765\frac{Btu}{lb_m \cdot °F}\right)\Delta T = 2.5 \times 10^6 \frac{Btu}{hr}$$

$$\Delta T = 8.6°F$$

$$\Delta T = T_2 - T_1 = T_2 - 32°F = 8.6°F$$

$$T_2 = 40.6°F$$

Having established the exit temperature for the glycol mixture, draw the two streams of the parallel flow heat exchanger and calculate the LMTD.

$$Hot\,Fluid:\;65°F \longrightarrow 55°F$$

$$Cold\,Fluid:\;32°F \longrightarrow 40.6°F$$

Define one *physical* side of the heat exchanger as 'A' and the other side as 'B' and determine the respective temperature differences.

$$\Delta T_A = 65°F - 32°F = 33°F$$

$$\Delta T_B = 55°F - 40.6°F = 14.4°F$$

Use the formula below to calculate the log mean temperature difference.

$$LMTD = \frac{\Delta T_A - \Delta T_B}{ln\left(\frac{\Delta T_A}{\Delta T_B}\right)}$$

$$LMTD = \frac{33°F - 14.4°F}{ln\left(\frac{33°F}{14.4°F}\right)} = 22.4°F$$

Write an expression for the heat transfer of the heat exchanger as a whole, isolate area, and solve.

$$\dot{Q}_{hx} = UA\Delta T_{lm}$$

$$A = \frac{\dot{Q}_{hx}}{U\Delta T_{lm}} = \frac{2.5 \times 10^6 \frac{Btu}{hr}}{\left(60\frac{Btu}{hr \cdot ft^2 \cdot °F}\right)(22.4°F)} = 1858ft^2$$

Answer A